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A STUDY OF THE SEMIANNUAL DENSITY VARIATION IN THE UPPER ATMOSPHERE FROM 1958 TO 1966, BASED ON SATELLITE DRAG ANALYSIS

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ABSTRACT

The semiannual variation has been isolated in the drag data of six artificial satellites with perigee heights ranging from 250 to 658 km in the interval from 1958 through 1966 by suppression of all other known types of upper atmosphere variation with the help of empirical formulas. Although the shape of the curve is somewhat variable from year to year, the semiannual oscillation is a very stable feature and can be followed without any major change in phase throughout the 8 years covered by the observations. The temperature curves obtained from each of the six satellites are strictly in phase and show the same amplitude, irrespective of perigee height; peculiarities of the variation, such as an unusually broad minimum or a late maximum, are easily recognizable in each of the individual satellite curves. This shows that the semiannual variation is worldwide and that the observed density variations are the result of temperature variations at essentially the same atmospheric level as those arising from the solar-activity effect. All previously known features of the semiannual variation, such as the systematic inequality in the maxima and minima and the proportionality between the amplitude of the temperature variation and the 10.7-cm solar flux, are confirmed.

RÉSUMÉ

La variation bi-annuelle a été isolée dans les données de trainée de six satellites artificiels de hauteur de périgée allant de 250 à 658 km et pour une période allant de 1958 à 1966. Les résultats furent obtenus en éliminant au moyen de formules empiriques tous les autres types connus de variations dans la haute atmosphère. Bien que la forme de la courbe soit quelque peu variable d'une année à l'autre, la variation bi-annuelle est une caractéristique très stable qui se poursuit sans changement de phase important tout au long des 8 années d'observation. Les courbes de température obtenues pour les six satellites sont strictement en phase et ont la même amplitude, quelle que soit la hauteur de périgée; dans chacune des courbes relatives aux différents satellites on reconnait facilement certaines particularités de la variation telles qu'un minimum exceptionnellement large ou un maximum tardif. Ceci montre que la variation bi-annuelle est un phénomène mondial et que les variations de densité observées sont le résultat de variations de température qui ont lieu essentiellement au même niveau atmosphérique que les variations dues à l'effet de l'activité solaire. Toutes les caractéristiques déjà connues de la variation bi-annuelle, telles que l'inégalité systématique entre les maxima et les minima ou la proportionnalité entre les variations de température et le flux solaire 10,7 cm, sont confirmées.

KOHCHEKT

Полугодовое изменение было выделено в данных драга шести искуственных спутников с высотами перигей колеблющимися между 250 и 658 км.в интервале от 1958 до 1966 г., изымая все прочие известные типы изменения высшей атмосферы с помощью эмпирических формул. Несмотря на то что форма кривой до некоторой степени изменяется из года в год, полугодовое колебание является очень постоянным качеством и может быть следимым без какого либо существенного изменения в фазе за 8 лет в течение которых производились наблюдения. Кривые температуры, полученные от каждого из шести спутников являются точно в фазе и указывают одинаковую амплитуду независимо от высоты перигея; особенности изменения, как необычно широкий минимум или запоздалый максимум, легко распознаваемы в каждой из отдельных спутниковых кривых. Это указыват на то что полугодовое изменение является мировым и что наблюдаемые изменения плотности происходят как результат изменений температуры на в общем, одинаковых атмосферических уровнях как и те что происходят от эффекта солнечной деятельности. Все ранее знаемые характеристики полугодового изменения, таковые как систематическая неравность максимумах и минимумах и пропорциональность между амплитудой изменения температуры и 10,7 см. солнечным потоком были утвержденны.

A STUDY OF THE SEMIANNUAL DENSITY VARIATION IN THE UPPER ATMOSPHERE FROM 1958 TO 1966, BASED ON SATELLITE DRAG ANALYSIS

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1. THE SEMIANNUAL VARIATION

The semiannual variation is the least understood among the several types of variation that are observed in the upper atmosphere. Every year, the atmospheric density above 200 km reaches a deep minimum in July, followed by a high maximum in October to November; these are followed by a secondary minimum in January and a secondary maximum in April. Paetzold and Zschörner (1960, 1961), who first called attention to the phenomenon, named it the "plasma effect," in the belief that its cause had to be sought in the interaction between the solar wind and the magnetosphere, modulated by the orbital motion of the earth. They broke down the variation into a semiannual and an annual component and suggested that the latter could be caused by the interstellar wind. Jacchia (1965a) found that the observed density variations could be explained by temperature variations in the thermosphere and that the amplitude of the temperature variations is proportional to solar activity and, more specifically, to the intensity of the 10.7-cm solar flux. Since this is true of both the semiannual and the annual components, he concluded that these were not separate phenomena, but rather characteristics of a single type of variation. An attempt by Anderson (1966a, b) to explain the semiannual variation as an illusion caused by the motion in latitude of satellite perigees has been effectively disproved by King-Hele (1966a, b), who showed that the semiannual variation is conspicuously present even in the drag of polar satellites in circular orbits. Cook and Scott (1966) and Cook (1967) found that at a height of 1100 km the semiannual variation around sunspot minimum is too large to be explained

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entirely by temperature variation in the thermosphere and suggested that there could be, in addition, parallel variations in the height of the turbo-pause for helium. Jacchia (1967), however, believes that the discrepancy is only apparent, being caused by an error in the hydrogen content of the comparison models. A semiannual variation in the ionosphere, parallel to that of the neutral atmosphere, has been recognized for some time, both in the effective height (Becker, 1966) and in the electron density (Radicella and Cosio de Ragone, 1966; Yonezawa, 1966) of the F2 peak.

2. METHOD OF ANALYSIS

For the present analysis we have assembled all available drag data for six satellites, from among those tracked by the Baker-Nunn cameras of the Smithsonian Astrophysical Observatory, that are particularly suited for a study of the semiannual variation. The first criterion for the selection of these satellites was that they should have been observed without serious interruptions for several cycles of the variation. From this selection, however, we had to eliminate those satellites with high inclinations and perigee heights above 500 km, because of the difficulty of accounting with a sufficient degree of accuracy for the effect of the winter helium bulge (Jacchia and Slowey, 1967; Keating and Prior, 1967). Basic data for the six selected satellites appear in Table 1.

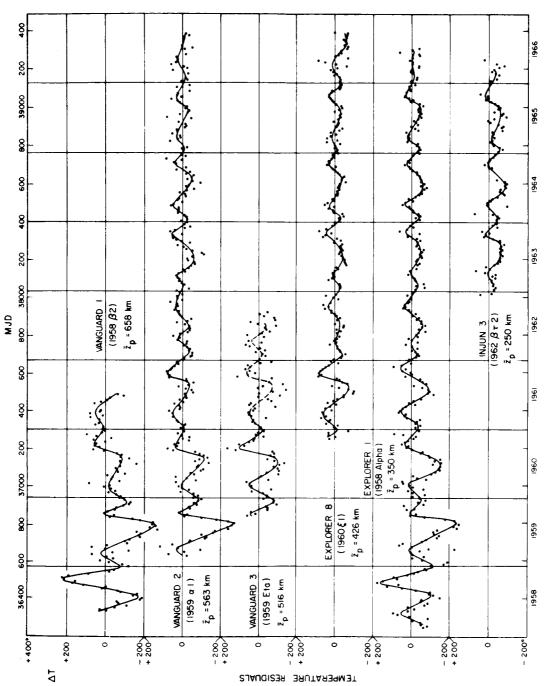
Table 1. Characteristics of satellites

	Z p (km)	i	Observations
Vanguard 1 (1958 β2)	658	34 . 3	May 1958 — July 1961
Vanguard 2 (1959 al)	563	32.9	March 1959 — November 1966
Vanguard 3 (1959 Eta)	516	33.3	September 1959 — September 1962
Explorer 8 (1960 \$1)	426	49.9	October 1960 - November 1966
Explorer 1 (1958 Alpha)	350	33.2	February 1958 — July 1966
Injun 3 (1962 βτ2)	250	70.4	December 1962 — July 1966

We isolated the semiannual variation by the same method and the same computer program that had been used by Jacchia and Slowey (1967) to isolate the diurnal variation. We eliminated the solar-activity effect, the diurnal variation, and the geomagnetic effect by using the equations given by Jacchia (1965b), with the following modifications:

- A. The value of the diurnal-variation factor R in equations (10) and (11), in agreement with the recent results by Jacchia and Slowey (1967), was taken to be 1.32 for all dates before February 1963, and 1.26 from August 1963 onward; in between these dates, R was made to decrease linearly.
- B. The coefficient of the exponential function in equation (14), which accounts for the geomagnetic effect, was decreased from 125° to 100°, in agreement with the recent results by Jacchia, Slowey, and Verniani (1967). In addition, the time lag in the atmospheric variations with respect to those in the geomagnetic index was changed to 0.279 days, or 6.7 hours.
- C. A time lag of 1.5 days was introduced in the solar-activity effect, as given by equation (8), in agreement with the recent results by Roemer (1967).

As in the analysis of the diurnal variation, we used 10-day means of the temperature residuals. We gave each individual observation a weight proportional to the interval of differentiation of the positional data from which the orbital accelerations (and thus the temperatures) were derived. This procedure minimizes possible errors introduced by inadequate correction for magnetic-storm activity. The 10-day means thus obtained are shown separately for each of the six satellites in Figure 1; a summary of the maxima and minima from these curves is given in Table 2.



of empirical formulas. MJD in abscissa is the Modified Julian Day (J. D. minus 2400000.5). in decreasing order of perigee height. Plotted are the 10-day means of the temperature residuals obtained when all other known atmospheric variations are suppressed by means The semiannual temperature variation from the drag of six artificial satellites, arranged Figure 1.

Observed maxima and minima of the semiannual variation normalized to $\overline{\mathbf{F}}_{10.7}$ = 100 Table 2.

Year d* ΔT AT ΔT AT <t< th=""><th></th><th>Sec</th><th>Secondary minima</th><th>minima</th><th>Se</th><th>condary</th><th>Secondary maxima</th><th>d.</th><th>Primary minima</th><th>ninima</th><th>ď</th><th>Primary maxima</th><th>naxima</th></t<>		Sec	Secondary minima	minima	Se	condary	Secondary maxima	d.	Primary minima	ninima	ď	Primary maxima	naxima
- -	1	*•		ΔT (<u>F</u> _{10.7} =100)	**0		$\overline{(\overline{\mathbf{F}}_{10.7=100})}$			$(\overline{\mathbf{F}}_{10.7}^{\Delta T})$	* p	ΔΤ	$(\overline{F}_{10.7}^{\Delta T})$
5d -90° -39° 93 +25 +12 229 -248 -113 -14 -82 -43 71 +13 +7 193 -113 -68 2 -14 -12 90 +51 +49 219 -62 -54 38 -32 -32 108 +6 +5 187 -38 -44 26 -27 -36 86 +18 +22 203 -42 -56 26 -27 -36 95 +35 +49 210 -44 -64 24 -21 -29 84 +20 +27 213 -34 -45 30 -12 -15 110 +10 +11 - - - - 21 -15 16 +10 +11 - - - - - 21 -17 91 +25 210 - - - - 21 -17 40 +20 +27 210 -	8	١	١	1	116 ^d	+61°	+26°		-132°	-58	p 562	+169°	+77°
-14 -82 -43	6	5 ^d	.06-	-39° (-31)	93	+25	+12 (+35)	622	-248	-113	285	+13	+7 (+54)
2 -14 -12 90 +51 +49 219 -62 -54 38 -32 -32 108 +6 +5 187 -38 -44 31 -18 -22 86 +18 +22 203 -42 -56 26 -27 -36 95 +35 +49 210 -44 -64 24 -21 -29 84 +20 +27 213 -34 -45 30 -12 -15 110 +10 +11 - - - - 4Jan. 23 -23 -90 +29 +29 (July 30) -50 4Jan. 26 -23 90 +29 (July 25) -50	09	-14	-82	-43	7.1	+13	+7 (+39)	193	-113	-68	281	+46	+30 (+38)
38 -32 -32 108 +6 +5 187 -38 -44 31 -18 -22 86 +18 +22 203 -42 -56 26 -27 -36 95 +35 +49 210 -44 -64 24 -21 -29 84 +20 +27 213 -34 -45 30 -12 -15 110 +10 +11 - - - 21 -17 91 +25 210 - - - (Jan. 25) -23 -23 90 +29 -59 -50 (Jan. 26) -23 -23 -24 -46 -50 -50	51	2	-1.4	-12	06	+51	+49	219	-62	-54	307	+ 68	+74
31 -18 -22 86 +18 +22 203 -42 -56 26 -27 -36 95 +35 +49 210 -44 -64 24 -21 -29 84 +20 +27 213 -34 -45 30 -12 -15 110 +10 +11 - - - 4 Jan. 23 -23 90 +29 (July 30) -50 4 Jan. 26) -23 40 +29 (July 25) -50	29	38	-32	-32	108	9+	+5	187	-38	† †	285	+36	+43
26 -27 -36 95 +35 +49 210 -44 -64 24 -21 -29 84 +20 +27 213 -34 -45 30 -12 -15 110 +10 +11 - - - 21 -17 91 +25 210 -50 (Jan. 22) -23 90 +29 (July 25) -50 (Jan. 26) (Apr. 1) (Apr. 1) -29 -50	53	31	-18	-22	98	+18	+22	203	-42	-56	311	+40	+52
24 -21 -29 84 +20 +27 213 -34 -45 30 -12 -15 110 +10 +11 - - - 21 -17 91 +25 210 -50 (Jan. 22) -23 90 +29 205 -50 (Jan. 26) -23 90 +29 (July 25) -50	54	97	-27	-36	96	+35	+49	210	-44	-64	311	+33	+46
21 -17 91 +25 210 -50 (July 30)	65	24	-21	-29	84	+20	+27	213	-34	-45	562	+34	+44
21 -17 91 +25 210 -50 (Jan. 22) (Apr. 2) (July 30) -50 (July 30) -50 (Jan. 26) (Apr. 1) (Apr. 1) (July 25) -50	99	30		-15	110	+10	+11	I	ı	I	ı	1	I
25 -23 90 +29 205 -50 (Jan. 26) (Apr. 1)	1958 to 1966	21 (Jan. 22)		-17	91 (Apr. 2)		+25	210 (July 30)		-50	299 (Oct. 27)		+49
	1961 to 1965	25 (Jan. 26)		-23	90 (Apr. 1)		+29	205 (July 25)		-50	304 (Nov. 1)		+49

The figures in parentheses are values corrected for the dip in the mean curve observed in 1959 to 1960 (see text and Figure 2) * d = days since January 1

3. RESULTS

An inspection of Figure 1 immediately reveals the following facts:

- A. The semiannual variation is a very stable feature and can be followed without any major change in phase throughout the interval of more than 8 years covered by the observations.
- B. The shape of the temperature curve is somewhat variable from year to year.
- C. The individual temperature curves obtained from each of the six satellites are strictly in phase and show the same amplitude, irrespective of perigee height. Any peculiarity of the curve such as a maximum or minimum that is exceptionally high, or broad, or late, etc. can be recognized in all the curves that cover that particular time interval.
- D. The amplitude of the variation was large at sunspot maximum (1958 to 1959) and decreased toward sunspot minimum (1963 to 1965).

During the years 1959 and 1960, the mean yearly residual was strongly negative for the four satellites that were observed at that time. We believe this can be explained by a temporary failure of the 10.7-cm solar flux to represent correctly the intensity of the heating radiation during that interval; in other words, the relation between atmospheric temperature and the 10.7-cm flux cannot be considered perfect — a result that was to be expected.

In each of the diagrams of Figure 1 the line $\Delta T = 0$ corresponds to zero residual from the Jacchia (1965b) models. We see that the average of the year is very close to zero for the years 1961 to 1966 at the heights corresponding to 1959 al (516 km) and 1960 ξ 1 (426 km). The curve of 1958 Alpha (350 km) is systematically 10° lower, and that of 1962 $\beta\tau$ 2 (250 km) shows a mean residual of -23°. This indicates that the models predict densities that are a little higher: by 0.02 in log ρ at 350 km and by 0.03 at 250 km (5% and 7%, respectively). Although this discrepancy could be real, its smallness

does not preclude the possibility that it might arise from an uncertainty in the area-to-mass ratio of the satellites or from a moderate height dependence of the drag coefficient.

An intercomparison of the residuals for different satellites leads to the individual systematic residuals shown in Table 3.

Table 3. Systematic residuals $\overline{\Delta T}$ from the Jacchia (1965b) models

Satellite	\overline{Z}_{p}	$\overline{\Delta ext{T}}$
1958 β2	658	- 5
1959 al	563	0
1959 Eta	516	+14
1960 \$1	426	-3
1958 Alpha	350	-10
1962 βτ2	250	-23

The systematic residual for 1959 Eta must not be considered seriously, since the presentation area for this satellite of rather complicated shape is not accurately known. Moreover, the data from this satellite are affected by larger scatter because the effect of solar-radiation pressure on it could not be accurately computed on account of its odd shape; this situation led to its finally being dropped from the observing list, when atmospheric drag became comparable in magnitude with radiation-pressure effects.

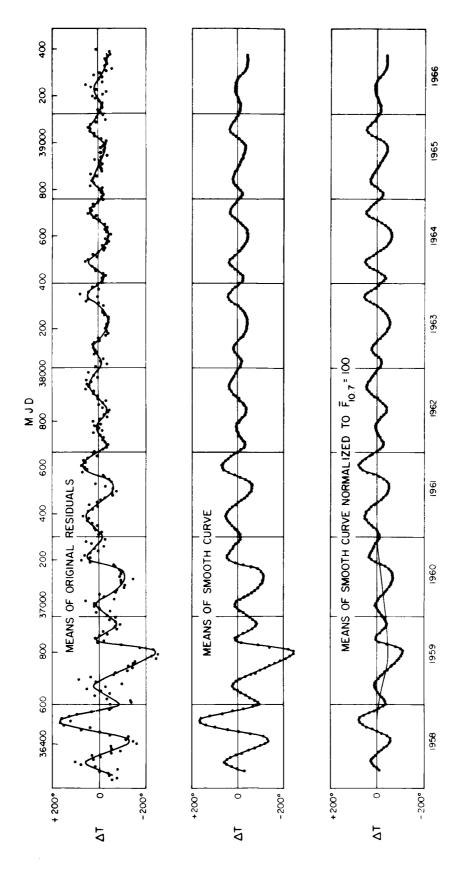
In combining the data from different satellites, we have taken into account the systematic residuals of Table 3; in addition, we have used the weights, given in Table 4, that reflect the degree of accuracy of the observational data.

Table 4. Weights used for the computation of mean residuals

Satellite	Time interval (MJD)	Weight
1958 β2	Beginning to 37000	2
	37000 to 37488	1
	After 37488	0
1959 al	Beginning to 38200	2
	After 38200	1
1959 Eta	Beginning to 37400	1
	After 37400	0
1960 \$1		2
1958 Alpha		2
1962 βτ2		1

Mean curves of the semiannual variation from 1958 to 1966 are shown in Figure 2. We computed the mean residuals in two different ways: First, we computed the means using the original observations, which are shown in the top diagram. Then we computed the means of the ordinates read off at 10-day intervals on the smoothed curves of Figure 1. The result (second diagram in Figure 2) is an array of points lying on a smooth curve, which the eye can follow more easily. The same curve, of course, fits the more scattered points of the top diagram. Means of smooth curves were used in an earlier analysis of the semiannual variation (Jacchia, 1965a); here we have given means of both the original and the smoothed data to show that they lead to identical results.

In the last diagram in Figure 2, we have plotted the smoothed residuals of the center strip after reducing them to $\overline{F}_{10.7}$ = 100, assuming that the amplitude of the semiannual effect is proportional to $\overline{F}_{10.7}$. As can be seen, this assumption is justified, inasmuch as the amplitude of the curve seems



the smooth curve dipping below the zero line in 1959 to 1960 is drawn through the average satellites in Figure 1. Plotted in the top diagram are the weighted means of the original 10-day means of the temperature residuals. The dots in the second diagram are the Figure 1. The last diagram shows the data from the preceding strip normalized to $10.7 = 100 \ (\overline{\rm F}_{10.7} \ \rm is$ the smoothed 10.7-cm solar flux, in units of $10^{-22} \ \rm W/m^2 c/sec)$; Mean curves of the semiannual variation obtained by combining the data from the six weighted means of the ordinates read off at 10-day intervals on the smooth curves of level of the temperature residuals during those 2 years and probably represents a temporary failure in the relation between the temperature and \overline{F}_{10} . 7. Figure 1. Figure 2.

now to be statistically the same from sunspot maximum to sunspot minimum. In this last diagram, we have also drawn a smooth curve through the average level of the temperature residuals in the years 1959 to 1960, when, as already mentioned, it differed considerably from the zero line.

The data of the second diagram of Figure 2 are replotted in Figure 3, year by year, to show at a glance the changes in the semiannual variation, not only in amplitude and shape of the curve, but also in the time of maxima and minima.

We then proceeded to derive an average annual curve of the semiannual variation normalized to $\overline{F}_{10..7}$ = 100. In addition to averages over the whole interval (1958 to 1966) covered by the observations, we have derived means for the 5 years from 1961 through 1965, eliminating the incomplete years 1958 and 1966 and the years 1959 to 1960, in which the mean level of the variation departed from the zero line. In taking the 1958 to 1966 means, we applied corrections to the observations of the years 1959 to 1960, equal to the ordinates of the smooth curve in the bottom diagram of Figure 3, with changed signs. The results are seen in Figure 4, where again we have used the original observations and the data from the smooth curves. The same curve has been drawn for reference in all four diagrams to show the small systematic difference between the general 1958 to 1966 means and those of the interval 1961 to 1965; this curve is the one that best fits the points on the first diagram.

The systematic difference between the 1958 to 1966 data and those for the interval from 1961 to 1965 is small enough to justify the adoption of the curve derived from the longer interval as a standard. In Table 5, we have assembled, in the first of the four data columns, the original ΔT means of the 1958 to 1966 period, computed from curve data. Although these means are reasonably smooth, we have, for interpolation purposes, smoothed them still further by use of graduation formulas: These smoothed means appear

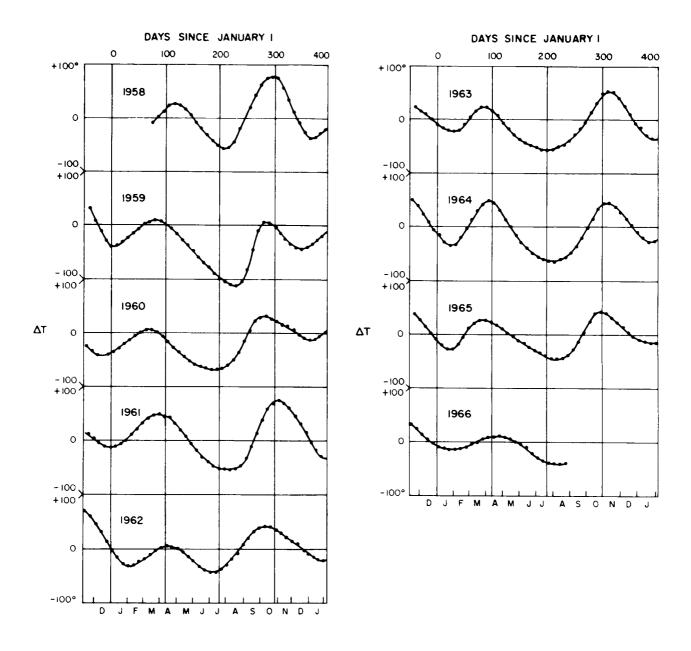
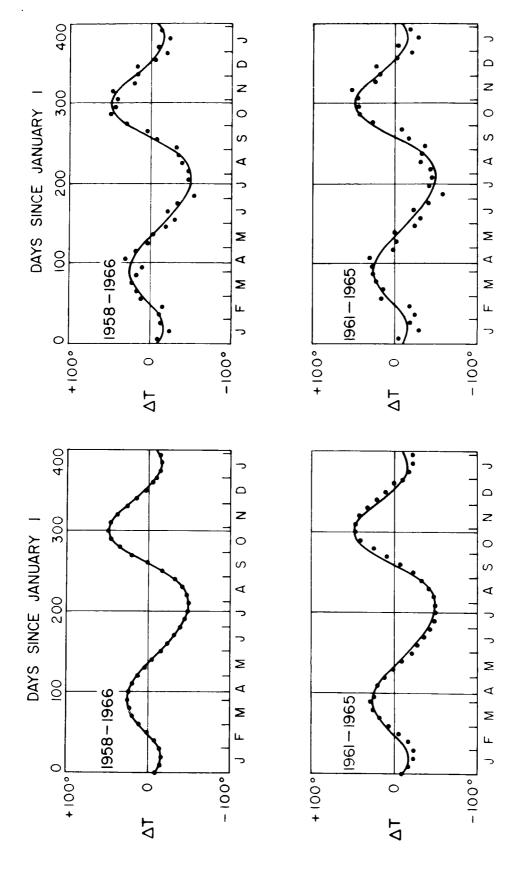


Figure 3. The semiannual variation, year by year. The data are the same as those in the middle strip of Figure 2, but aligned year by year to show phase differences.



been repeated for reference in the other three to reveal systematic differences in the means. Mean values of the semiannual variation normalized to $\overline{\mathrm{F}}_{10..7}$ = 100 (see legend to Figure 2). The dots in the two diagrams in the left half of the figure were obtained by averaging obtained by averaging the original, unsmoothed data after normalization and applying the same correction in 1959 to 1960. The curve that fits the points on the first diagram has mean level in 1959 to 1960; those in the two diagrams in the right half of the figure were the data shown in the last of the three strips of Figure 3, corrected for the dip in the Figure 4.

Table 5. Observed and computed mean values of the semiannual temperature variation, normalized to $\overline{F}_{10.7}$ = 100

		1 9	1958 to 1966		1961 to 1965		19	1958 to 1966	99	1961 to 1965
Date		ΔT observed	ΔT curve	ΔT computed	ΔT observed	Date	ΔT observed	ΔT curve	ΔT computed	ΔT observed
		(1)	(2)	(3)	(1)		(1)	(2)	(3)	(1)
Jan.	_	-9:2	-9.1	-11:6	-7:6	July 9	-45,3	-44.9	-43°6	-47:8
	11	-14.5	-14.5	-15.6	-16.6	19	-48.3	-48.7	-47.9	-50.2
	21	-16.5	-16.3	-15.4	-22.0	29	-50.0	-49.9	-50.1	-49.8
	31	-14.5	-14.1	-11.9	-22.2	Aug. 8	-48.2	-48.2	-48.8	-47.2
Feb.	10	-8.4	-8.0	-6.5	-16.0	18	-42.6	-42.6	-42.9	-42.4
	20	+0.9	+1.2	+0.1	-5.0	28	-33.0	-32.0	-31.9	-34.6
March	2	+11.0	+10.5	+7.8	+7.6	Sept. 7	-17.4	-16.6	-16.4	-22.8
	12	+19.5	+18.2	+16.2	+17.8	17	+1.6	+1.7	+1.7	-7.2
	22	+22.2	+23.1	+23.5	+26.0	27	+21.1	+20.3	+19.7	+9.6
April	-	+24.9	+24.8	+27.5	+29.0	Oct. 7	+35.9	+35.5	+34.9	+26.0
	1.1	+24.1	+23.5	+26.7	+27.6	17	+45.6	+45.2	+45.1	+40.2
	21	+19.3	+19.4	+21.1	+20.8	27	+48.9	+48.4	+49.0	+48.4
May	-	+12.6	+12.7	+12.5	+12.2	Nov. 6	+46.0	+45.6	+46.7	+48.6
	1.1	+4.1	+4. 0	+2.7	+2.2	16	+38.0	+38.0	+39.2	+43.0
	21	-5.1	-5.4	-7.1	-8.6	56	+27.0	+27.3	+28.0	+33.6
	31	-15.8	-15.0	-16.0	-20.8	Dec. 6	+15.5	+15.6	+15.1	+22. 4
June	10	-23.8	-24.1	-24.1	-28.0	16	+4.5	+4.4	+2.5	+10.8
	20	-32.1	-32.2	-31.3	-36.2	56	-5.2	-5.1	-7.7	-0.6
	30	-39.2	-39.2	-37.8	-42.8					

(1) Means of yearly normalized curves.

⁽²⁾ Smooth curve through points of preceding column.

⁽³⁾ Computed from equation (1).

in the second data column headed "curve." For the benefit of those who prefer to use an analytical function instead of tabular values, we submit the following expression:

$$\Delta T = 2.41 + \overline{F}_{10.7}[0.349 + 0.206(2\pi\tau + 226.5)] \sin(4\pi\tau + 247.6)$$
, (1)

where

$$\tau = \frac{d}{Y} + 0.1145 \left\{ \frac{1 + \sin[2\pi(d/Y) + 342^{\circ}3]}{2} \right\}^{2.16} - \frac{1}{2}$$

In this formula, d is the number of days elapsed since January 1 and Y is the tropical year, in days. Values of ΔT computed with this formula are given in the third data column of Table 5. In the last column are the means, from curves, of the 1961 to 1965 interval. The dates of maxima and minima of the semiannual variation and the corresponding values of ΔT from curves of the 1958 to 1966 and 1961 to 1965 periods and from equation (1) are given in Table 6.

Table 6. Mean maxima and minima of the semiannual variation normalized to $\overline{F}_{10.7}$ = 100

	1958 to 1960	1961 to 1965	Equation (1)
m'	January 22	January 26	January 15
$\Delta \mathrm{T}$	-17°	-23°	-16°
M'	April 2	April l	April 3
ΔT	+25°	+29°	+28°
m	July 30	July 25	July 30
ΔT	-50°	-50°	-50°
M	October 27	November l	October 28
ΔT	+49°	+49°	+49°

Lastly, to check for a possible dependence of the amplitude of the semi-annual variation on height, we have computed means, normalized to $\overline{F}_{10..7}$ = 100, from the data of each satellite, separately; 1959 Eta was omitted because the data were insufficient to secure a reliable mean curve. The results are shown in Figure 5, where again the satellites are arranged in decreasing order of height. As can be seen, there is no clear evidence of a systematic change of the amplitude with height, and we must conclude that the observed density variations are the result of temperature variations at essentially the same level as in the case of the solar-activity effect. On the other hand, the fact that the shape of the curve is the same, no matter from which satellite it is derived, shows that the semiannual variation affects the whole atmosphere in the same manner, irrespective of latitude.

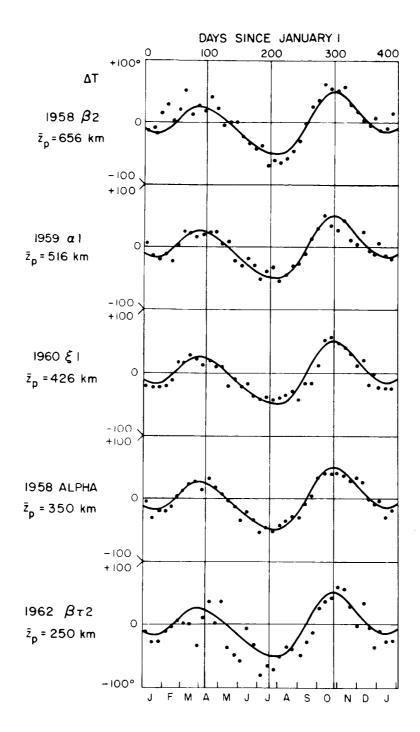


Figure 5. Means of the semiannual variation, reduced to $\overline{F}_{10.7}$ = 100 (see legend to Figure 2), computed separately from the data of five satellites (those of 1959 Eta were excluded because they covered only a relatively short time interval). The means of the unsmoothed 10-day means are plotted. The same smooth curve that was used for reference in Figure 4 is also used here in all diagrams.

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BIOGRAPHICAL NOTES

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NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

First issued to ensure the immediate dissemination of data for satellite tracking, the reports have continued to provide a rapid distribution of catalogs of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals. The Reports are also used extensively for the rapid publication of preliminary or special results in other fields of astrophysics.

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